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Renegotiation of Troubled Debt: The Choice between Discounted Payoff and Maturity Extension

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Renegotiation of securitized debt contracts is generally a more efficient solution to default than foreclosure when there are significant deadweight costs associated with the enforcement of security rights. Recent literature shows that when renegotiation takes the form of discounted loan payoffs, it eliminates deadweight costs associated with the liquidation or transfer of assets. There is evidence, however, that, in practice, renegotiation of other contract terms such as maturity is a more common form of loan workout. This observation is puzzling because, in general, maturity renegotiation does not eliminate deadweight costs. We provide a partial answer to this puzzle by showing that maturity renegotiation better aligns the incentives of borrowers and lenders than does renegotiation of principal. Specifically, we find that borrowers who expect that lenders will renegotiate maturity in the event of default have less incentive to divert cash flow from the collateral during the term of the loan and less incentive to take on additional risk. If the lender's cost of managing these standard agency problems is positively related to the magnitude of the borrower's incentive, then maturity renegotiation will result in lower monitoring and enforcement costs.

Renegotiation of debt contracts is a tool that borrowers and lenders have long used to improve the efficiency of financial contracts. Renegotiation is preferable to strict enforcement of the lender's right to the borrower's assets when value would be lost in the process of transferring and liquidating pledged assets.[1] For example, real estate lenders frequently renegotiate or "work out" problem loans because of the significant time and costs associated with the foreclosure process.

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[1] The rationale for renegotiation rather than including provisions in the original loan contract that specify outcomes in the event of financial distress is the cost of writing complex contracts and the limited enforceability of contracts (see Huberman and Kahn 1988).

Deadweight foreclosure costs can result in inefficiency in debt contracts. The possibility of deadweight costs means that borrowers, *a priori*, place a higher value on the required loan payments than lenders place on the net payments they receive. Consequently, rational borrowers reject loans that generate zero profits for lenders, and profitable investment opportunities are lost. If borrower and lender expect that negotiation in the event of financial distress will eliminate these deadweight costs, then the inefficiency is eliminated.[2]

Recent literature (*e.g.*, Riddiough and Wyatt 1994a, Anderson and Sundaresan 1996, Mella-Barral and Perraudin 1997) shows that when renegotiation takes the form of discounted loan payoffs (renegotiation of principal), it eliminates all deadweight costs associated with the liquidation of assets. For example, in the Mella-Barral and Perraudin model, defaulting borrowers make "take-it-or-leave-it" discounted payoff offers to lenders equal to the net proceeds lenders expect to receive after transfer. These offers are acceptable to lenders and beneficial to borrowers. In equilibrium, all occurrences of financial distress are resolved via discounted loan payoffs and the contracting inefficiency is eliminated.

There is substantial evidence, however, that renegotiation of other contract terms is more common in loan workouts than discounted payoffs (see Asquith, Gertner and Scharfstein 1994 and Mann 1997). These authors find that renegotiation of maturity is the most common form of loan modification. Asquith, Gertner and Scharfstein study companies in financial distress, and report that although the banks in their study almost never accepted discounted loan payoffs, they frequently modified other loan terms, including maturity. Mann reports that discounted payoffs occur in only a small fraction of the problem loan workouts he studied, while other forms of contract modification are common.[3] Maturity extension is especially prevalent in real estate lending. For example, many servicing agreements associated with Commercial Mortgage-Backed Securities (CMBS) give the special servicer the right to extend maturity to resolve a

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[2] The willingness to renegotiate can induce additional defaults by borrowers who are not experiencing financial distress but want to use the renegotiation process to extract concessions from lenders. See Riddiough and Wyatt (1994a) for a more complete discussion of these "strategic" defaults. The increased frequency of default does not, of itself, generate an inefficiency. Inefficiency is a function of the possibility of deadweight costs, not the possibility of default. When the form of renegotiation eliminates all deadweight costs, the cost of these additional strategic defaults can be priced in the original loan contract without loss of contractual efficiency.

[3] Mann (1997) studies 72 troubled debt cases selected from the portfolios of a finance company, a bank, and an insurance company. In the sample of 21 insurance company defaults, 3 (14%) were resolved by discounted payoffs, while 12 cases(57%) were resolved by modifying other elements of the loan contract. Ten of these 12 included maturity extension.

default but not the right to forgive principal.[4] Fannie Mae, the world's largest mortgage investor, rarely accepts discounted payoffs from defaulting borrowers, but frequently provides relief via maturity extension.[5] These findings and observations raise an interesting question: Why do market participants forego the renegotiation mechanism that has been shown to eliminate deadweight costs and use maturity renegotiation instead?

In this paper, we argue that a partial answer to this question lies in the fact that the expectation of maturity renegotiation can reduce two well-known agency problems associated with debt contracts. Gertner and Scharfstein (1991) characterize the two agency problems as underinvestment and overinvestment. The underinvestment problem is based on Myers' (1977) argument that because default truncates the borrower's interest in the collateral, the borrower does not fully value the benefits of investment in the property and may forego positive net present value investment opportunities. On the other hand, Jensen and Meckling (1976) show that the asymmetry of the borrower's "call-like" payoff can encourage excessive risk taking. For corporate borrowers, this risk-taking can be accomplished by undertaking negative net present value projects that have a wide dispersion of outcomes. In this paper, we show that borrowers who expect that lenders will resolve default through renegotiation of maturity rather than renegotiation of principal have less incentive to underinvest and less incentive to overinvest. Lenders attempt to limit these agency problems through loan covenants and costly monitoring and enforcement. Adopting a workout strategy that reduces the borrower's incentives to underinvest and overinvest should reduce the lender's cost of dealing with these standard agency problems.

While these principles apply to any borrowing relationship where the borrower's future actions influence the market value of the underlying collateral, they are especially relevant in real estate lending because of the combination of high loan-to-value ratios, substantial borrower discretion over the use of cash flows from the property during the term of the loan, and high foreclosure or deadweight costs. In the context of real estate lending, the underinvestment problem can be viewed as one of requiring the borrower to maintain the property properly. The real estate equivalent to the overinvestment problem is the incentive

[4] Riddiough (1997b) discusses the use of maturity extension as a workout mechanism in asset-backed securities. In the context of asset-backed securities with both junior and senior claimants, junior security holders will generally prefer maturity extension over discounted payoffs because of their first loss position. Riddiough concludes that, in general, junior security holders should control the renegotiation process. He also points out that maturity extension can be viewed as a "volatility increasing" action from the perspective of the senior debtholders. In this paper, we focus on the original borrower and lender.

[5] Source: private conversations with Fannie Mae loan servicing staff.

for the borrower to adopt operating practices, such as altering the tenant mix or the form of the leases, that increase the volatility of the market value of the pledged assets.

We first calculate the expected payoff to the borrower as a function of the collateral value at loan maturity, conditional on the form of renegotiation (*i.e.*, maturity extension or discounted payoff). We then analyze how differences in expected payoff functions affect the borrower's incentive to maintain the property and/or alter the volatility of the market value of the property. We find that when the borrower expects default to be resolved via maturity extension rather than discounted payoff, the borrower has less incentive to underinvest. Our findings with respect to overinvestment depend on the cash dividend rate of the property. For moderate dividend rates (*i.e.*, rates slightly above the risk-free rate, or less), we find that maturity renegotiation reduces the incentive to add risk. For dividend rates well in excess of the risk-free rate, we find the reverse true. Because dividend yields vary by property type,[6] these findings suggest that the optimal form of renegotiation may vary systematically by property type. Long-run commercial real estate dividend payout rates (relative to the risk-free rate) suggest that, in general, maturity renegotiation reduces the incentive to increase the volatility of the property value. Overall, our results suggest that lenders who use maturity renegotiation to resolve default should incur lower agency costs[7] related to monitoring and enforcing loan covenants.

Maturity renegotiation does not eliminate the incentive to underinvest—it only reduces the incentive relative to the level that would prevail if discounted payoffs were used to resolve default. While other factors such as accounting requirements, capital regulations, and taxes may influence the lender's choice of workout form, the analysis of these factors is beyond the scope of this paper. The finding that the form of renegotiation affects the borrower's incentive to maintain the value of the asset over the life of the loan provides a new insight on loan contracting.

The paper is organized as follows. The next section develops a single-period model of maturity renegotiation and presents closed-form results for the

[6] For example, Pagliari *et al.* (2001) report that over the period from 1979 to 1998, apartments in the NCREIF database had an average dividend yield of 7.2% compared with 5.2% for regional malls.

[7] Agency costs associated with monitoring and enforcing loan covenants are effectively deadweight costs and have the same effect on contractual efficiency as do foreclosure costs. However, to avoid confusion, we use the term deadweight costs in the paper to refer to the costs associated with a lender enforcing its right to take control of and sell the pledged property. We use the term agency costs to refer to the costs associated with writing, monitoring, and enforcing loan covenants.

efficiency (with respect to the elimination of deadweight costs) of both maturity and principal renegotiation. The third section analyzes the incentive to underinvest associated with the different forms of renegotiation, and the fourth section analyzes the corresponding incentive to overinvest.

Single-Period Maturity Renegotiation

We view the renegotiation process as a two-player, noncooperative game played by the lender and borrower. We study the case where, at most, a single one-period maturity extension can be negotiated. If an extension is negotiated, then no further renegotiation of any of the loan terms is permitted at the end of the extension. While it is clear that a single, one-period maturity extension will not eliminate deadweight costs,[8] it is important to quantify the effectiveness in a way that facilitates comparison of maturity extension with discounted payoffs in the following sections.

General Assumptions

We consider nonamortizing[9] loan contracts that are collateralized by the pledge of commercial real estate. The loan is made at $t = 0$ and the loan maturity is denoted by T. A balloon payment (including accrued interest), L, is due at T. The underlying asset value, $V(t)$, follows a standard lognormal diffusion process:

$$\frac{dV}{V} = (\mu - \xi)\,dt + \sigma_V\,dZ.^{10} \tag{1}$$

The asset pays a cash dividend at a fixed rate ξ. The drift, $(\mu - \xi)$, of the asset price process represents the expected instantaneous rate of return from price appreciation of the asset. In subsequent sections, we assume that the borrower can influence, within limits, the dividend payout rate, ξ, and the volatility, σ_V. For

[8] As long as transfers of assets from borrowers to lenders occur, there will be deadweight costs. Thus, as long as there is a possibility that the asset value after the extension will be less than the required loan payoff, there will be a nonzero probability of a transfer of assets and deadweight costs.

[9] The loan contract can require either periodic payments of interest at a specified rate or the accrual of interest (*i.e.*, a zero coupon loan). If interest payments are required during the loan term, we assume they do not trigger default before maturity. The assumption of nonamortizing loans simplifies the model development without greatly distorting the cash flows. Most commercial real estate loans are structured with 25- or 30-year amortization schedules and maturities of 10 years or less. These "balloon" loans require lump sum payments at maturity of 75%–90% of the original loan amount (depending on the balloon date, the amortization period, and the coupon rate on the loan).

[10] The literature on commercial real estate has consistently estimated the volatility parameter σ_V to fall in the range from 0.15 to 0.225. See, for example, Titman and Torous (1989) or Childs, Ott and Riddiough (1996, 1997). Commercial property returns published by NCREIF suggest a cash dividend rate of 0.05 to 0.10.

example, a borrower can increase ξ by reducing maintenance expenditures or increase σ_V by altering the mix of tenants. In the following sections, we use ξ to measure underinvestment and σ_V to parameterize the overinvestment problem.

We assume that defaults occur only at loan maturity, T.[11] If a borrower defaults, then the lender has the right to take control of the pledged assets and sell them.[12] We assume that when lenders sell a property with market value $V(T)$, they are only able to recover $\gamma V(T)$, where $\gamma < 1$. The assumption, $\gamma < 1$, is justified by transaction costs such as sales commissions, legal fees, and lack of expert knowledge about managing the property. Basically, anything that results in a lender being able to generate net proceeds less than the current fair market value of the property in the hands of its current owner should be included in the discount. Obtaining a good point estimate of γ is difficult because the relevant discount should be measured against the value of the property if there had not been a foreclosure.[13] Empirical studies of commercial mortgage foreclosures only observe the transaction price and probably underestimate γ. Our interpretation of the available empirical studies of direct and indirect bankruptcy and foreclosure costs for commercial real estate loans (*e.g.*, Snyderman 1991, 1994, Fitch Investors Service 1996, 1998, Ciochetti 1997, Esaki, L'Heureux and Snyderman 1999) suggest that a reasonable range for γ is {0.70–0.90}.[14] We assume that both borrower and lender know γ.

[11] This assumption is reasonable for most commercial real estate loans where a typical 10-year maturity and 30-year amortization schedule result in a substantial balloon payment at maturity.

[12] Many financial institutions (*e.g.*, commercial banks) make loans secured by collateral in which the institution cannot invest directly. For example, banks commonly lend on the security of real estate but cannot invest directly in real estate. Such lenders must sell the assets shortly after taking ownership through foreclosure.

[13] For example, a property in the process of foreclosure may lose tenants that it would have otherwise retained. Similarly, a foreclosing lender is at a disadvantage in negotiating lease terms because of the uncertainty surrounding future ownership.

[14] The two Fitch studies of commercial mortgage default from CMBS issued in the period from 1991 through 1996 provide the most detail on the components of loss. In the most recent study, Fitch Investors Service (1998) reports that property protection expenses were 7.7% of the loan balance at default. In addition, losses resulting from the decline in property values accounted for 35.8% of the loan amount. This loss component includes selling costs such as broker commissions, marketing costs, legal fees, and transfer taxes. If such sales costs are estimated to be 6%–10% of the loan amount (Ciochetti 1993 provides some empirical evidence on lender cost magnitudes), the total deadweight costs would be 13.7% to 17.7%. Synderman (1991, 1994), Esaki *et al.* (1999), and Ciochetti (1997) analyze the default experience of loans originated by life insurance companies. The two Snyderman studies report total expenses associated with foreclosure of 27% to 33% of the loan amount, but they do not provide details. Ciochetti reports that direct foreclosure costs (legal and filing fees and other costs associated with a property during the foreclosure process) averaged 2.6% of the loan balance for loans that defaulted and were foreclosed between 1986 and 1995. However, Ciochetti analyzes costs only

Renegotiation Terms

At maturity, we assume borrowers and lenders act to maximize the current market value of their expected payoffs. Because the borrower can choose to default or make the required payment, she is the first mover in the renegotiation. At T, the borrower has two options: pay the loan in full and take unencumbered ownership of the asset or default. When a borrower defaults, she specifies the terms under which she is willing to extend the deadline for making the required payment, L, for one period. We use a single payment, b, from borrower to lender to represent these terms. If the borrower defaults (and makes an offer to extend), then the lender must choose between two courses of action: foreclose and sell the asset or accept the offer for a one-period extension of the loan. When developing their strategies, both parties know that no further renegotiation can occur after the extension. With full, symmetric information the borrower can anticipate the lender's choice when deciding to pay off the loan or default. Consequently, the borrower knows the minimum acceptable offer, b, and will only make an offer that she knows will be accepted. After an extension, the borrower defaults when $V(T+1) < L$ and makes the required loan payment when $V(T+1) > L$.

Loan workout agreements can be quite complex and involve numerous covenants and agreements between borrower and lender. For example, the borrower may be required to make payments to the lender during the extension period, the final loan payment can differ from L or the borrower may be required to invest in the assets securitizing the loan. For example, Mann (1997) found that in two-thirds of the cases where loan terms were relaxed, borrowers were required to invest additional capital in the underlying assets. In order to avoid modeling all such variations, we parameterize the terms of the extension using only the initial payment, b, required from the borrower at the time of the extension. The economic effect of varying the other terms is measured in present value and viewed as equivalent to changes in b. In addition, we assume:

1. All cash flow from the underlying asset during the extension is reinvested in the asset—that is, $\xi = 0$. This "no dividend" assumption means the asset drift equals μ during the period of extension.[15]

through the technical completion of foreclosure of the mortgage contract and is thus an incomplete measure for our purposes. Although the best point estimate of γ based strictly on these studies would be approximately 0.85, we have chosen a somewhat larger range of 0.7 to 0.9 to be consistent with previous literature (*e.g.*, Riddiough and Wyatt 1994b use a range of 0.75 to 0.90) and to facilitate sensitivity analysis.

[15] This assumption is consistent with normal loan workout practice. For example, loan covenants generally require (at least after a delinquency) that all cash flows pass through a lender-controlled "lock box" with only residual cash flows passed through to the borrower.

Table 1 ■ Normal form of the noncooperative game.

Lender Strategy	Borrower Strategy: Default and Offer \$b for an Extension	Full Payoff of the Loan
Foreclose	$0 \mid \gamma V(T)$	$V(T) - L \mid L$
Accept	$\mathrm{C}(V(T), L, r, \sigma_V, t_P) - b \mid V(T) - C_m(V(T), L, r, \sigma_V, t_P, \gamma) + b$	$V(T) - L \mid L$

Table 1 is the normal form of the noncooperative game played by the borrower and lender at loan maturity T. The entries in the cells are: borrower payoff | lender payoff. $V(T)$ represents the asset value at T. γ is the percentage of the asset value recovered by the lender if assets are transferred. $\mathrm{C}(V(T), L, r, \sigma_V, t_P)$ is the value of a call option on V when the current asset value is $V(T)$, the strike price is L, the interest rate is r and the asset variance is σ_V. C_m is the "modified" call option value described in Equation (5). The modified call differs from a standard call by incorporating the market value of expected deadweight costs that the lender will incur if the call option is not exercised.

2. The loan payoff required after the extension is L. Ignoring the payment, b, the loan extension is interest-free with no adjustment in required loan payment.[16]

3. The risk-free rate of interest, $r(T)$, is observed by both borrower and lender at T and remains constant over the extension period. The level of interest rates at the time of the extension influences the optimal strategies.

Strategies

Both borrower and lender know that when renegotiation is not possible (such as at the end of the extension, $T + 1$), as long as more is preferred to less, rational borrowers will make the required payment, L, at maturity when the value of assets at risk exceeds the required loan payoff ($V(T + 1) > L$). Default and costly liquidation will occur when the asset value is less than L.

Table 1 presents the normal form for the noncooperative game described by these assumptions. Each column (row) of the table corresponds to a choice by the borrower (lender). Each cell presents the payoffs to the borrower and lender,

[16] Viewing L as the principal amount of the borrower's obligation, lowering L is equivalent to a discounted payoff—and is excluded here by assumption. Any increase in L can be captured by b. For example, an offer to make interest payments at the original loan rate during the extension would result in b equal to the expected present value of the additional interest payments.

with the borrower's payoff listed first. The borrower has two possible strategies at T: full payoff of the loan or default and offer \$$b$ to extend. The lender's two possible responses are to either foreclose or to accept the offer for extension.

The entries in the table for the column labeled "Full Payoff of the loan" are standard borrower/lender payoffs. The borrower retains the asset by paying L, thereby receiving a net payoff of $V(T) - L$. The lender receives the required loan payoff, L. The possible payoffs if the borrower defaults are given in the column labeled "Default and Offer \$$b$ for an Extension." If the borrower makes an offer to extend and the lender accepts, then the borrower receives a call option (with a strike price, L, and a life, t_p[17]) on the asset. We denote the market value of the call as $C(V(T), L, r, \sigma_V, t_p)$, or simply C when suppressing the standard arguments will not cause confusion. Under the general assumptions described above, the value of C is given by the standard Black–Scholes (1973) option formula. After considering the initial payment, b, to the lender, the borrower's payoff is $C - b$.

The lender's claim, when there is an extension, includes the following elements:

1. A long position in the asset: $V(T)$.
2. A short position in a "modified" call on the asset: $C_m(V(T), L, r, \sigma_V, \gamma, t_p)$.
3. The payment b.

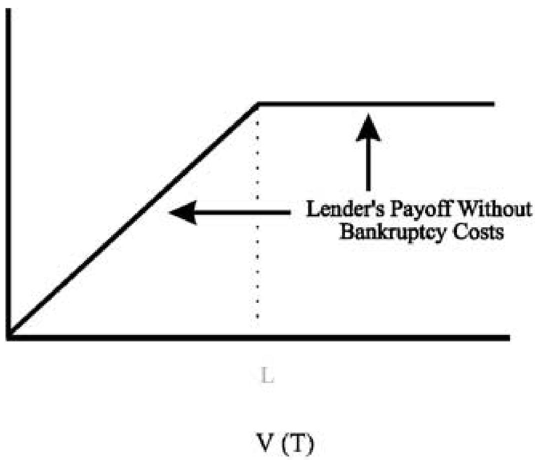
The modified call represents a claim whose payoff is equal to the combination of the payment of deadweight costs when the borrower does not exercise the call and the standard payoff to a call (with strike price L) when exercise is optimal. The rationale for combining these two payoffs into a single claim is that the lender does not care to whom a payment is made; the lender cares only about its own net payoff from the transaction.

Figure 1 contrasts the lender's claim without deadweight costs (Panel A) with the payoff when there are deadweight costs (Panel B). In Panel A, the lender's payoff equals the payoff from being long the asset and short a standard call on that asset with strike price equal to L. When proportional deadweight costs associated with the lender taking control of the asset are included, the lender's payoff is reduced relative to that shown in Panel A by a "wedge" of costs paid to third parties when $V(T + 1) < L$. The lower envelope of the lines in Panel B depicts this altered payoff pattern. We combine the two reductions (relative to a

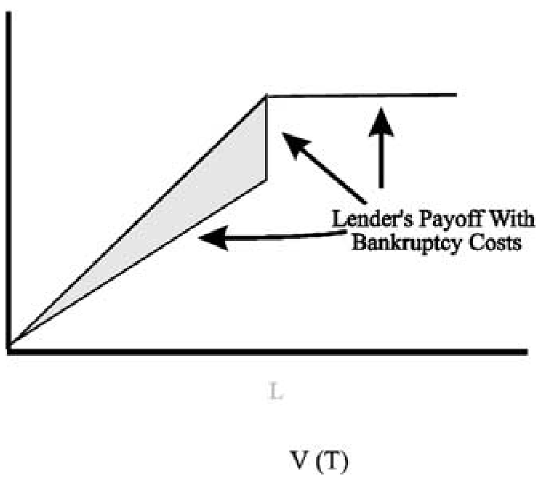
[17] In this section, we focus exclusively on one-period extensions, and therefore $t_p = 1$.

Figure 1 ■ Lender's payoff functions.

Panel A: Without deadweight costs.



Panel B: With deadweight costs.



simple long position in the asset) into a single contingent claim—the modified call option. We denote the wedge payment (*i.e.*, the shaded area in Panel B) by $W(V(T), L, r, \sigma_V, \gamma, t_p)$, or W. The modified call, $C_m(V(T), L, r, \sigma_V, \gamma, t_p)$ (C_m) is equal to $C + W$ and measures the cost to the lender of foregoing $V(T+1) - L$ when asset values are high and $(1-\gamma)V(T+1)$ when asset values are low.

Appendix A shows that under these assumptions,

$$W(V(T), L, r, \sigma_V, \gamma, t_p) = (1-\gamma)V(T)\Phi(-z_1) \quad (2)$$

where $\Phi(-z_1)$ represents the cumulative normal distribution function evaluated at $-z_1$ and z_1 is defined as in the Black–Scholes formula.[18]

Using Equation (2) and substituting the Black–Scholes formula for $C(V(T), L, r, \sigma_V, t_p)$ into the expression $C_m = C + W$ gives

$$C_m = V(T)\Phi(z_1) - Le^{-rt_p}\Phi(z_2) + (1-\gamma)V(T)\Phi(-z_1),$$
$$\text{where } z_2 = z_1 - \sigma_V\sqrt{t_p}. \tag{3}$$

The lender's total payoff, $V(T) - C_m + b$, is then:[19]

$$\gamma V(T)\Phi(-z_1) + Le^{-rt_p}\Phi(z_2) + b. \tag{4}$$

Completing Table 1, if the borrower defaults and the lender forecloses, then the borrower's payoff is zero and the lender receives $\gamma V(T)$.

The lender's optimal response to default by the borrower is determined (by dominance) from the relative payoffs in the default column. Extension is preferred when $V(T) - C_m + b > \gamma V(T)$.

Using the expression in (4), extension is preferred when

$$b > \gamma V(T)\Phi(z_1) - Le^{-rt_p}\Phi(z_2). \tag{5}$$

As first mover, the borrower must decide whether to pay off the loan or default and make the lender an offer. From Table 1, we see that if b is acceptable to the lender and still below the premium over parity[20] of the option received, the borrower will prefer default.

To better illustrate the borrower's decision, Figure 2 plots a function equal to the right-hand side of Inequality (5), labeled "Lender's Indifference

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[18] $z_1 = \frac{\ln\left(\frac{V(T)}{L}\right) + \left(r + \frac{\sigma_V^2}{2}\right)t_p}{\sigma_V\sqrt{t_p}}$.

[19] Longstaff (1990) derives an equivalent closed-form expression for the gain from debt extension in the presence of deadweight bankruptcy costs. Longstaff discusses the potential gain from extending the maturity of defaultable debt but does not contrast that gain with the potential gain from discounted payoff of the loan, nor does he consider the possible effects on borrower behavior during the original term of the loan.

[20] The premium over parity is the difference between the market value of the option and the "intrinsic" value of immediate exercise.

Figure 2 ■ Lender's indifference payment and borrower's payoffs.

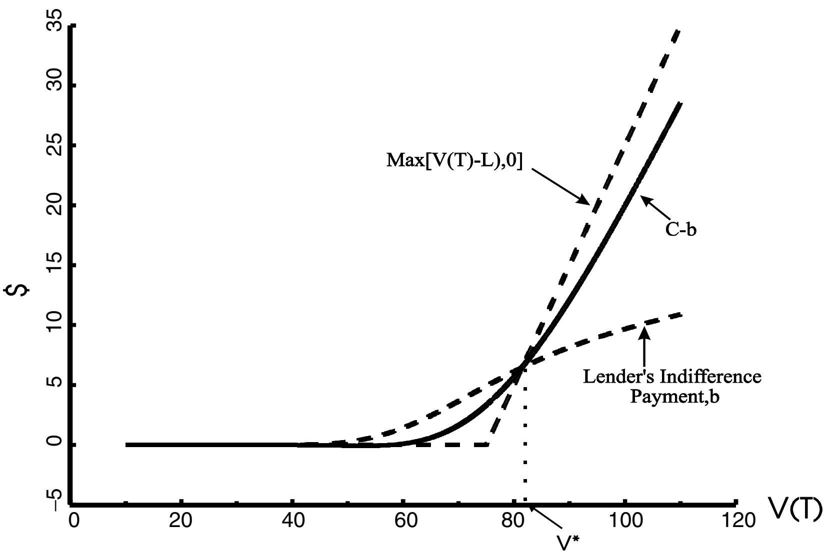

Figure 2 displays the relationship among the lender's indifference payment, the standard payoff function for a borrower (Max[$V(T) - L$, 0]) and the net benefit to a borrower of an extension if the borrower must make a payment, b, to obtain the extension. The lender's indifference payment represents the size of the payment, b, from the borrower to the lender that would make the lender just indifferent between agreeing to a one-period extension and terminating the loan agreement at $t = T$ by transferring the underlying assets. The standard borrower payoff function reflects the fact that, at maturity, the borrower has a call option on the underlying assets with strike price equal to the required loan payment. The curve labeled $C - b$ shows the payoff to the borrower from a one-period extension of the loan. The borrower would obtain a new one-period call option with value C in return for making a payment to the lender sufficient to assure the lender's willingness to extend (*i.e.*, b). To the right of V^*, the standard loan payoff exceeds the payoff from an extension and the borrower will pay off the loan. To the left of V^*, the borrower will default and offer the lender b for an extension. As long a b equals or exceeds the indifference amount, the lender will agree to the extension. The line labeled "$C - b$" sets b equal to the lender indifference payment.

Payment."[21] If b is set marginally above this level, then the lender will accept the offer to extend. Knowing this, the borrower sets b at the level indicated by this function and compares Max[$V(T) - L$, 0] with $C - b$. These two functions are also shown in Figure 2. The figure shows that for high asset values (*i.e.*, greater than V^* where ($C - b$) and Max[$V(T) - L$, 0] intersect), the lender requires a large payment, b, to accept an extension and with b set to the indifference level,

[21] The parameter values used to generate the figure are $\sigma_V = 0.2$, $r = 0.06$, $L = 75$, $\gamma = 0.9$, and $t_p = 1$.

$C - b < \text{Max}[V(T) - L, 0]$. The borrower's optimal choice in this region is to make the required loan payment, L. To the left of the critical value, V^*, the borrower's optimal strategy is to default and offer the lender the minimum payment required to ensure acceptance.[22] If b is set equal to the lender's indifference payment, then the function $C - b > 0$[23] and is preferred to $V(T) - L$ for low values of $V(T)$.

For fixed asset process parameters and a fixed interest rate, the borrower's strategy can be completely described using the critical value, V^*, where $C - b = \text{Max}[(V(T) - L), 0]$. If $V(T) > V^*$, then the optimal strategy is to pay off the loan. For $V(T) < V^*$, the optimal strategy is to default and offer a payment equal to the lender's indifference payment. As long a $\gamma < 1$, $V^* > L$. (See Appendix B.) As a result, no costly transfers of assets will occur at T–all cases of financial distress (*i.e.*, $V(T) < L$) will result in renegotiation and extension. However, deadweight costs are not eliminated. Given the asset process in Equation (1), $\text{Prob}[V(T+1) < L \mid V(T) < V^*] > 0$ and so, on average, costly transfers will occur at $T + 1$.[24]

Efficiency

The top panel of Table 2 shows the payoffs to borrower and lender when the lender is willing to accept a discounted payoff (and the borrower has sufficient liquidity to make the payment). As before, renegotiation occurs at loan maturity and is conditioned on the asset value at maturity, $V(T)$. This form of renegotiation is efficient because whether the borrower defaults and negotiates a discounted payoff or makes the required loan payment, the sum of the two

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[22] The lender's indifference payment drops below zero, indicating that the lender is willing to pay the borrower to extend the loan when the market value of uncertain future deadweight costs is less than the certain costs that will be incurred if the lender forecloses now.

[23] Setting b equal to the lender's indifference payment shows that $C - b = (1 - \gamma)V(T) - W$. Substituting for W from Equation (2) shows that $C - b = (1 - \gamma)V(T)\,[1 - \Phi(z_1)] > 0$.

[24] If V^* were equal to L, then the outcome of the noncooperative game would minimize the incidence of deadweight costs for a world where renegotiation was restricted to a single one-period maturity extension. It can be shown, however, that $V^* > L$ (see Appendix B). Because $V^* > L$, there will be extensions based on strategic defaults (*i.e.*, defaults when $V(T) > L$) as well as extensions resulting from financial distress ($V(T) < L$). Some of the extensions from strategic defaults will end with costly transfers of assets that would have been avoided without renegotiation. The net benefit of introducing maturity renegotiation depends on the relationship between the number of strategic defaults that end with costly transfers and the number of cases of financial distress where costly transfer is avoided by the extension. Monte Carlo simulations show that when γ is significantly less than 1, strategic defaults account for as much as 40% of all defaults and the net benefit of renegotiation is small.

Table 2 ■ Comparison of maturity renegotiation with discounted payoffs.

	$V(T) < V^{**}$ Default and Renegotiate	$V(T) > V^{**}$ Full Payoff
Panel A: Lender and borrower payoffs with discounted payoff		
Borrower	$(1-\gamma)V(T)$	$V(T)-L$
Lender	$\gamma V(T)$	L
Total	$V(T)$	$V(T)$
Trigger level, V^{**}	L/γ	
Panel B: Lender and borrower payoffs with a single-maturity extension		
Borrower	$(1-\gamma)V(T)\Phi(z_1)$	$V(T)-L$
Lender	$\gamma V(T)$	L
Total	$V(T)[\gamma+(1-\gamma)\Phi(z_1)] < V(T)$	$V(T)$
Trigger level, V^{*}	$L < L/[\gamma+(1-\gamma)\Phi(-z_1) < L/\gamma$	

Panel A shows the payoffs to borrowers and lenders at the maturity date, T, of an original secured loan when renegotiation takes the form of negotiating a discounted payoff. $V(T)$ is the asset value observed at time T; L is the required loan payment due at T, the maturity date of the original loan. γ measures the deadweight costs associated with a transfer of assets from borrower to lender. V^{**} is the asset price that divides the default and payoff regions at time T. At T, we assume that borrowers move first in a two-party noncooperative game. If borrowers want to negotiate a discounted payoff, they offer the lender a payoff amount that just exceeds the net value the lender would receive after a transfer of the asset.

Panel B shows the payoffs to lender and borrower that result when renegotiation is restricted to a single maturity extension. $V(T)$ is the asset value observed at time T; L is the required loan payment due at T, the maturity date of the original loan. γ measures the deadweight costs associated with a transfer of assets from borrower to lender. V^{*} is the asset price that divides the default and payoff regions at time T. At T, we assume that borrowers move first in a two-party noncooperative game. If borrowers want to negotiate an extension, they offer the lender a payment, b, that just exceeds the lender's indifference value in return for a one-period loan extension. At the end of the one-period loan extension, the lender takes the asset if $V(T+1) < L$ and the borrower pays the loan in full if $V(T+1) > L$. Because the option is for one period, $t_P = 1$.

payoffs equals the observed asset value. The default point (V^{**}) is easily shown to be L/γ.

The lower panel of Table 2 is based on the analysis in the "Strategies"subsection and shows the payoffs to borrower and lender if renegotiation is limited to a single one-period extension of maturity. In this case, when there is a default and renegotiation, the sum of the lender's and borrower's payoffs is $V(T)[\gamma + (1-\gamma)\Phi(-z_1)] < V(T)$. The critical value of $V(T)$ that triggers default and renegotiation is shown in Appendix B to be $V^{*} = L/[\gamma + (1-\gamma)\Phi(z_1)]$. With maturity extension, the trigger level for default, V^{*}, is lower (*i.e.*, closer to L)

than the trigger level for discounted payoffs. Both V^* and V^{**} are greater than L, and, as a result, both forms of renegotiation lead to strategic defaults—defaults that would not occur if the lender always refused any form of renegotiation.[25] However, because $V^* < V^{**}$, strategic defaults are less frequent with maturity renegotiation than with principal renegotiation.

Borrower-Expected Payoffs for Different Forms of Renegotiation

Figure 3 portrays the differences in borrower payoffs resulting from the different forms of renegotiation. The top panel displays the borrower's payoff as a function of the terminal value of the asset if no renegotiation is allowed. The borrower receives nothing when $V(T) \le L$ and receives $(V - L)$ when $V(T) > L$. The middle panel describes the borrower's payoff with discounted payoffs. First, the default trigger point is shifted to the right (to V^{**}). Further, when the borrower does default, she receives $(1 - \gamma)V_T$. The bottom panel describes the payoff function resulting from maturity renegotiation. To the left of V^*, the borrower receives $C - b$; to the right she receives $V - L$. From Appendix B, we know that $C - b$ is always less than $(1 - \gamma)V(T)$. It will be useful in the next section to use a single piecewise-linear function to approximate all three payoff patterns.

$$\text{Borrower's payoff function} = \left\{ \begin{array}{ll} (1-\gamma_k)V(T) & \text{for } V(T) \le \frac{L}{\gamma_k} \\ V(T) - L & \text{for } V(T) > \frac{L}{\gamma_k} \end{array} \right\}, \quad k = N, M, P, \tag{6}$$

N = No renegotion: $\gamma_N = 1$

P = Principal renegotiation: $1 > \gamma_P = \gamma$(the true recovery rate)

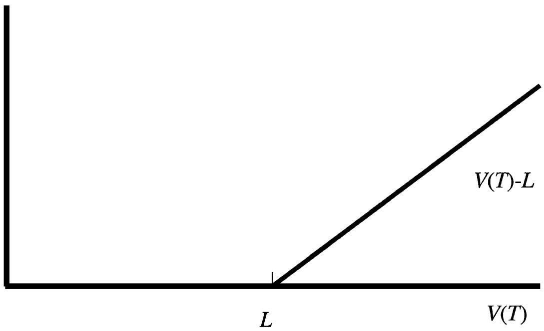
M = Maturity renegotiation: $1 > \gamma_M > \gamma_P$

This one-period maturity extension model can be extended to permit longer extensions or multiple extensions. If the renegotiation process was costless and an infinite number of renegotiation periods were permitted, then the payoff

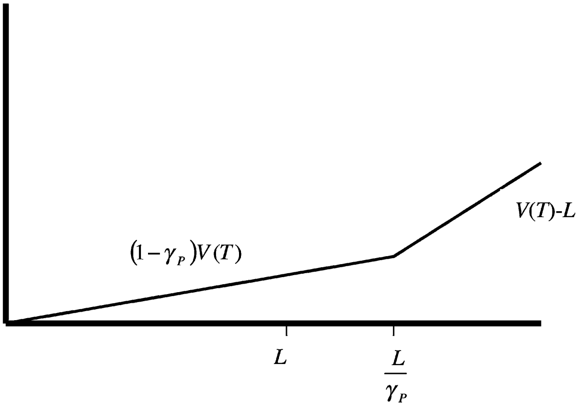
[25] See Riddiough and Wyatt (1994a, 1994b) for discussion of the lender's decision to renegotiate or foreclose. They find that in a repeated game, a variety of strategic outcomes can occur and provide an explanation for the coexistence of foreclosure and renegotiation.

Figure 3 ■ Borrower payoff at maturity under different forms of renegotiation.

Panel A: No Renegotiation.



Panel B: Principal Renegotiation.



Panel C: Maturity Renegotiation.

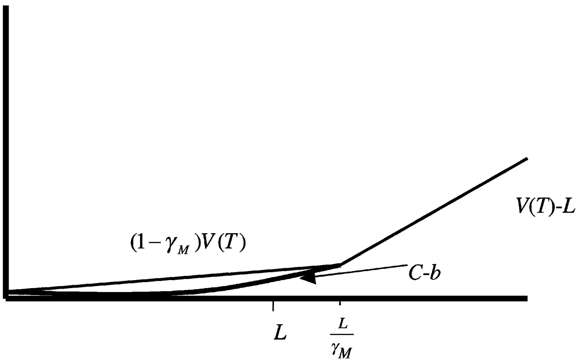


Figure 3 displays the borrower's payoff as a function of the asset value at time T for three different negotiation policies: A: No renegotiation; B: Principal renegotiation; C: Maturity renegotiation. In C, the true borrower payoff in the default region is $C - b$. The figure shows the linear approximation of Equation (6) that is used to calculate the incentives to underinvest and overinvest.

function would converge to that of principal renegotiation.[26] This result is consistent with standard option pricing results. For example, with constant interest rates and no dividends, a call option with no expiration has a value equal to the value of the underlying asset. With such a perpetual option, the lender is essentially transferring ownership of the asset to the borrower in return for the payment, b, which in the limit converges to $\gamma V(T)$.[27]

The Incentive to Underinvest

It has long been established (see Myers 1977 and Gertner and Scharfstein 1991) that loan contracts create incentives for borrowers to behave in ways contrary to the interest of the lender. In our model, the incentive to underinvest (*e.g.*, see Myers 1977) results in an incentive to undermaintain the property or equivalently to extract as much cash flow from the underlying property as possible. In terms of the model developed in the previous section, the incentive to underinvest can be treated as an incentive to increase the cash dividend rate, ξ.[28] Implicit in the results of the previous section was the assumption that the asset drift during the life of the original loan (and subsequent extension) was an exogenous constant, independent of the form of renegotiation. This is equivalent to assuming that the probability distribution describing asset values at T was fixed and independent of the form of renegotiation that borrower and lender expect. In this section, we assume that borrowers can select the value of ξ at the outset of the loan that maximizes the market value of their claims. We measure the incentive to underinvest by calculating the partial derivative of the market value of the borrower's claim with respect to the dividend payout rate, ξ.[29]

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[26] For example, consider the expressions in Table 2 when t_p increases toward infinity. From the borrower's payoff in Table 2, Panel B, we see that increasing $\Phi(z_1)$ reduces the deadweight costs. From the definition of z_1, we know that, for t_p sufficiently large, z_1 is an increasing function of t_p. When t_p approaches infinity, $\Phi(z_1)$ approaches 1 and the payoffs and the trigger level in Table 2, Panel B, converge to the corresponding values for discounted payoffs.

[27] The authors have also modeled the payoffs from repeated one-period extensions. While here, too, the maturity renegotiation payoff function converges toward that arising from principal renegotiation and deadweight costs are reduced, we conclude that the rate of convergence between the two forms of renegotiation is slow relative to the institutional requirements that lenders resolve problem loans expeditiously.

[28] The dividend rate is set at the time of loan origination and is not controlled dynamically over the life of the loan. Further, without introducing a specific cost function for reinvestment, our model provides a narrower concept of the underinvestment problem than that considered by Myers (1977). For example, in our model, the incentive to underinvest arises from the deadweight costs of foreclosure whereas the Myers model does not require deadweight costs.

[29] The methodology used here is similar to that used in Riddiough (1997a). In that paper, Riddiough analyzes the underinvestment problem by studying the incentive effects of debt financing on land investment, development decisions, and land valuation.

Evidence from defaulted loans suggests that commercial real estate borrowers have substantial flexibility in setting ξ. For example, in a recent study of the reasons for default, Fitch Investors Service (1999) reports that a common cause of default on multifamily loans is that borrowers "misdirected funds, failed to put money into the property or took money from the property" (p. 1). In the area of residential mortgage loans, Harding, Miceli and Sirmans (2000a, 2000b) show that homeowners with high loan-to-value ratios spend significantly less on maintenance of their homes.[30] These findings are both examples of borrowers reacting to the incentive to underinvest. As long as there is a possibility of default, the borrower does not receive the full benefit of an investment in the underlying collateral and rationally chooses to minimize that investment.

Lenders are aware of this incentive and protect their interest in several different ways. For example, loan covenants require borrowers to maintain the property, purchase insurance, and pay taxes. Some commercial loans require borrowers to escrow funds for future repairs and maintenance. Borrowers also are concerned with their reputations because they will likely need to borrow in the future and therefore act to limit property deterioration. While the continued functioning of the commercial real estate loan market is clear evidence that these protections are effective, it is also reasonable to assume that the cost of monitoring and enforcing these provisions is directly related to the magnitude of the borrower's incentive.

In the absence of any renegotiation (as in the top panel of Figure 3), the mortgagor can be viewed as holding a European call (C) on a dividend-paying asset (with strike price equal to the required loan payment, L) plus the right (R) to all dividends over the life of the call. The market value of both the call and the right depend on the dividend rate, ξ. Viewed as a function of the chosen fixed dividend rate,[31] the borrower's objective function is (the subscript N denotes no renegotiation):

[30] The possibility of default means that the benefits of higher future home values resulting from increased current maintenance is shared between borrower and lender while the cost of the maintenance is borne by the borrower alone. As a result, when rational borrowers choose a level of maintenance such that the marginal cost equals marginal benefit, those with lower benefits choose lower levels of maintenance. Harding *et al.* (2000a) confirm this prediction using data from the American Housing Survey. The authors also find (see Harding *et al.* 2000b) that state laws such as those prohibiting deficiency judgments also reduce the borrower's incentive to maintain.

[31] If the dividend rate is allowed to vary over time, with the borrower setting $\xi(t)$ based on $V(t)$, the problem becomes a stochastic control problem. The resulting Bellman equation can only be solved under very limiting assumptions about the parameters of the problem. Further, in order to avoid the corner solution, a nonlinear reinvestment cost function would have to be included.

$$\operatorname*{Max}_{\xi} P_N(\xi) = C(\xi) + R(\xi). \tag{7}$$

Straightforward calculation (using the standard Black–Scholes formula[32]) shows that

$$\frac{\partial P_N}{\partial \xi} = \tau V(0) e^{-\xi\tau}\left(1 - \Phi\left(z_1^N\right)\right) > 0, \tag{8}$$

where

$$z_1^N = \frac{\ln\left(\frac{V(0)}{K}\right) + \left(r - \xi + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}.$$

Viewing Equation (8) as the first order condition of the borrower's maximization problem, it is clear that there is no interior maximum. The borrower maximizes the market value of her position by setting ξ as high as possible. Increasing the dividend rate reduces the value of the borrower's call but increases the value of the right to the dividends. As long as $\Phi(z_1) < 1$ (*i.e.*, there is some chance of default), the benefit exceeds the cost.

Renegotiation of the loan contract alters the borrower's payoff pattern, as shown in Figure 3, and consequently alters the incentive to pay dividends. Consider first the case of discounted payoff. The borrower now receives $(1 - \gamma_P)V(T)$ when there is a default, and the default region is enlarged relative to the top panel. Using the representation of the different payoff patterns defined in Equation (6), the derivative of the borrower's portfolio with respect to ξ is (see Appendix C for details):

$$\begin{aligned}\frac{\partial P_i}{\partial \xi} &= V(0)e^{-\xi\tau}\tau\left[(\gamma_i - 1)\left(1 - \Phi\left(z_1^i\right) + n\left(z_1^i\right)\frac{\sqrt{\tau}}{\sigma}\right) - \Phi\left(z_1^i\right)\right] \\ &\quad + \ln(\gamma)\frac{\sqrt{\tau}}{\sigma} + \tau V(0)e^{-\xi\tau}, \quad (i = N, P, M),\end{aligned} \tag{9}$$

where $n(\bullet)$ is the probability density function for the standard normal distribution and

$$z_1^i = \frac{\ln\left(\frac{V(0)\gamma_i}{L}\right) + \left(r - \xi + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}.$$

[32] The value of the right to the dividends is simply the original asset value, $V(0)$ less the value of a European call with a strike price of zero or $V(0)(1 - e^{-}\xi\tau)$.

As would be expected from Figure 3, for $\gamma_i = 1$, (no deadweight costs), Equation (9) simplifies to Equation (8). To help understand the relationship between $\partial P_i/\partial\xi$ and γ_i, think of the borrower's payoff pattern (as shown in the top panel of Figure 3) as having two regions: the "default" region, where the borrower does not care about the terminal property value, and the "payoff" region, where the borrower's concern about terminal value is identical to that of a full equity owner. In the middle panel of Figure 3, because the borrower shares in the asset value in the default region, the borrower has an interest (albeit small) in the terminal value in that region. This effect reduces the borrower's incentive to extract dividends. Operating in the opposite direction, however, is the fact that the default region is enlarged because default now occurs as long as the asset value is less than L/γ.

Similarly, the sensitivity of $\partial P_i/\partial\xi$ to changes in γ_i has two components because γ_i enters Equation (9) both through the term $(\gamma_i - 1)$ and through z_1^i. Increasing γ_i reduces the borrower's concern with terminal value in the default region[33] (thereby increasing $\partial P_i/\partial\xi$), but also reduces the size of the default region (lowering $\partial P_i/\partial\xi$). For values of $\gamma_i \approx 0$, the default region is very large, and the former effect dominates. For values of γ_i in the range of 0.7 to 0.9, the reduction in the size of the default region dominates. Therefore, increasing γ_i reduces $\partial P_i/\partial\xi$. Figure 4 shows $\partial P_i/\partial\xi$ as a function of γ_i and ξ.[34]

Because the effect of a shift from discounted payoff to maturity extension can be characterized as an increase in γ, $0 < \partial P_M/\partial\xi < \partial P_P/\partial\xi$, for reasonable values of γ. The significance of the shift depends on the magnitude of γ_M relative to γ_D. Since $(1 - \gamma_M)V(T)$ represents a linear approximation of $(C - b)$, we use the expected value of $(C - b)$ to estimate the effective γ_M. Using reasonable values of the parameters,[35] the effective γ_M is 0.93 when $\gamma_P = 0.8$. Thus, Figure 4 shows that a borrower who expects the lender to deal with default by renegotiating maturity has significantly less incentive to extract cash from the underlying collateral during the loan term. The borrower's and lender's interests in the market value of the underlying collateral are better aligned.

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[33] A value of $\gamma = 0$ results in a slope of 1 on the value of $V(T)$ in that region, corresponding the slope of the payoff to an equity owner.

[34] The incentive to underinvest is also influenced by the initial loan-to-value ratio, the volatility of the property value, and the loan term. For very low loan-to-value ratios, the default region is small and the agency problems associated with risky debt are small. Consequently, the form of renegotiation has little effect on the borrower's incentive. For very high volatility, the peak of the surface in Figure 4 shifts to the southeast and the benefit associated with increasing γ is reduced. For shorter maturity loans, the importance of the dividend rate is reduced and the peak in Figure 4 becomes less diagonal. As a result, the significance of the benefit of maturity extension is greater at higher dividend rates.

[35] Specifically, $\sigma_V = 0.15$, $\gamma = 0.8$, $r = 0.06$ and $T = 10$ years.

Figure 4 ■ The borrower's incentive to underinvest.

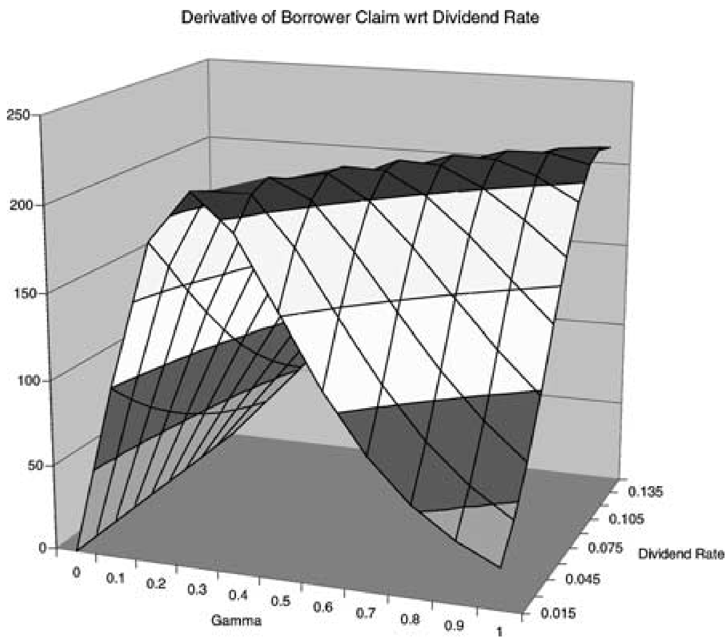

Figure 4 plots the borrower's incentive to undermaintain the property or, in general, underinvest (as measured by $\partial P_i/\partial P_i$ as a function of the true recovery rate, γ, and the dividend rate ξ. The other parameter values are $\sigma_V = 0.15$, $L = 80$, $V(0) = 100$, $r = 0.06$, and $\tau = 10$ years.

The Incentive to Overinvest

Jensen and Meckling (1976) point out that, in general, borrowers benefit by taking on more risk than lenders anticipated at the time of the loan contract. Gertner and Scharfstein (1991) characterize this as the incentive to overinvest because the incentive can lead borrowers to undertake new negative net present value projects that increase the overall riskiness of the enterprise. In real estate lending, this same phenomenon can be characterized as an incentive to increase the volatility of the asset price process of Equation (1), σ_V. In the absence of deadweight costs (or equivalently, when there is no renegotiation), the value of the borrower's portfolio, P_N, can be increased by increasing σ_V, thereby increasing the value of the borrower's call on the assets without reducing the expected value of the right to receive dividends. In this section, we analyze how the form of renegotiation influences the incentive to overinvest by looking at $\partial P_i/\partial \sigma$ for $i = N$, P, and M. As before, we facilitate the analysis by using the characterization of the different forms of renegotiation defined in Equation (6).

Figure 5 ■ The borrower's incentive to overinvest.

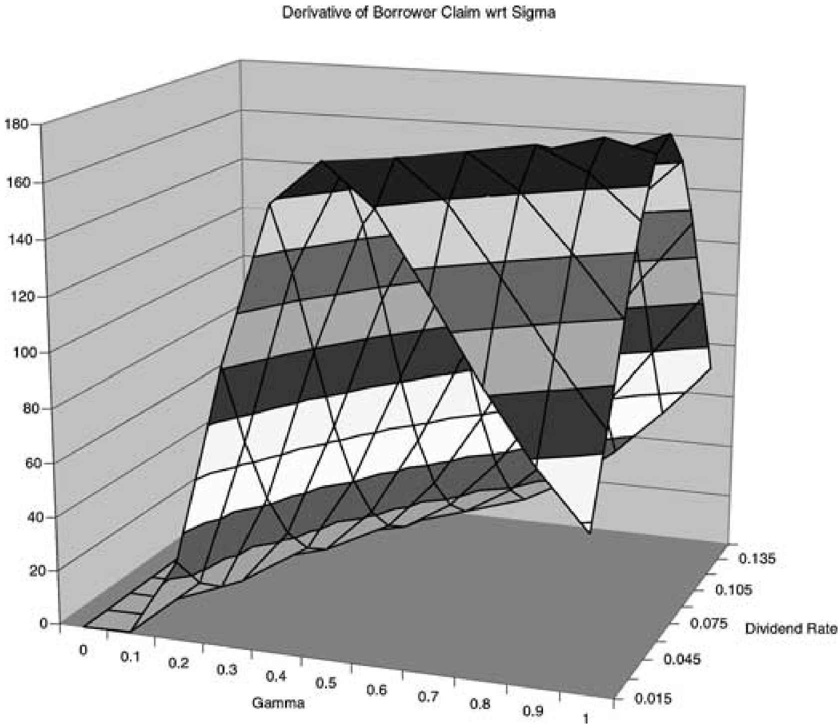

Figure 5 plots the borrower's incentive to take on additional risk or overinvest (as measured by $\partial P_i/\partial\sigma_V$) as a function of the true recovery rate, γ, and the dividend rate, ξ. The other parameter values are $\sigma_V = 0.15$, $L = 80$, $V(0) = 100$, $r = 0.06$, and $\delta = 10$ years.

As with the dividend payout rate, we assume the borrower can set σ_V at the outset of the loan contract and does not adjust it dynamically over time.[36] Once set, the volatility remains constant for the term of the loan. We calculate the "vega" of the borrower's portfolio by taking the partial derivative of the portfolio with respect to σ_V . The result is:

$$\frac{\partial P_i}{\partial \sigma_V} = (\gamma_i - 1)V(0)e^{-\xi\tau}n\left(z_1^i\right)\frac{\partial z_1^i}{\partial \sigma} + \left(V(0)e^{-\xi\tau}n\left(z_1^i\right) - Le^{-r\tau}n\left(z_2^i\right)\right)\frac{\partial z_1^i}{\partial \sigma} - Le^{-r\tau}n\left(z_2^i\right)\sqrt{\tau}. \tag{10}$$

[36] This simplification results in our model being a special case of the general overinvestment problem.

Equation (10) holds for $i = N, P$, and M. The third term in Equation (10) is the standard vega of a call option and results from Equation (10) when $\gamma_i = 1$.[37] For other values of γ_i, the resulting expression can be greater than or less than this base value. Figure 5 plots the expression as a function of both ξ and γ_i. In Figure 5, the incentive decreases with γ_i when the dividend rate is close to or less than the riskless rate (and γ_i is between 0.7 and 1). As in Figure 4, the relationship reverses when γ_i is low, but this is of little practical significance since lending contracts are very inefficient for low values of γ_i. In Figure 5, however, the negative relationship between γ_i and $\partial P_i/\partial\sigma_V$ disappears when the dividend rate is well above the risk-free rate. When the dividend rate is very high, increasing γ_i modestly increases the incentive to take on risk and principal renegotiation results in a lower incentive to take on risk. If the dividend rate is close to or below the risk-free rate, then renegotiation increases the incentive to take on risk (for reasonable values of γ_i) and maturity renegotiation is preferred over principal renegotiation. Long-run data from NCREIF suggest that, on average, the dividend payout rate for commercial real estate is only slightly above the risk-free rate, and so maturity renegotiation should reduce the incentive to overinvest. As with Figure 4, other parameters, such as loan-to-value ratio, loan term and volatility alter the shape of the surface of Figure 5, but do not change the general conclusion.[38]

Conclusion

This paper extends the literature on renegotiation of troubled debt by considering how the expected form of renegotiation influences borrower actions during the term of the loan. We find that borrowers who expect that lenders will renegotiate maturity in the event of default have less incentive to underinvest, that is, divert cash flow and value from the collateral during the term of the loan, than do borrowers who expect lenders to renegotiate the loan principal by accepting a discounted payoff. Our results with respect to overinvestment suggest that the optimal choice of renegotiation strategy depends on the characteristics of the particular property and loan. When lending on properties with moderate cash dividend rates (relative to the risk-free rate of interest), maturity renegotiation

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[37] The vega of a call option is frequently written as $V(0)e^{-\xi\tau}n(z_1^i)\sqrt{\tau}$, which is an alternative form of the third term in Equation (10).

[38] For very low loan-to-value ratios, the incentive to overinvest is small for all values of ξ and γ_i, and the form of renegotiation has little effect on the borrower's incentive. Increasing the property volatility shifts the peak in Figure 5 to the southeast, with the result that the range of dividend payout rates for which maturity extension is favored becomes smaller. For short maturity loans, the importance of the dividend payout rate is reduced and the peak in Figure 5 becomes more vertical and increases the range of dividend payout rates for which maturity extension is preferred. Extending the loan term has the opposite effect.

results in less incentive to take on new risks than does principal renegotiation. For properties with very high cash dividend rates, there is little difference between the two, but the ordering is reversed.

Overall, the standard agency results of Myers (1977) and Jensen and Meckling (1976) still apply. Borrowers have an incentive to extract as much cash as possible from the underlying collateral and to take on additional risk. Lenders must limit these actions through covenants and costly monitoring. If the cost of these lender actions is positively related to the magnitude of the incentive, then choosing the appropriate form of renegotiation will reduce the lender's cost of controlling these agency problems.

Because maturity renegotiation does not eliminate deadweight costs associated with foreclosure, the market's revealed preference for maturity renegotiation over principal renegotiation suggests that the benefit of reducing the agency problems (along with any accounting, regulatory, or other institutional factors) more than offsets the higher deadweight costs associated with maturity extension. Our results suggest that further research on troubled debt restructuring should consider the endogeneity of asset value drift and the potential influence that the expected form of renegotiation may have on asset values.

Empirical analysis of loan workouts could shed light on the relative importance of these incentives. For example, the model suggests that maturity renegotiation should be used more frequently for property types with low cash payouts such as regional malls or office properties. By studying loan workouts by a single lender with a varied portfolio of loans, one could hold institutional factors constant. In that case, a systematic variation in the type of workout used by property type would provide evidence supporting the significance of the incentive effect. On the other hand, empirical evidence that most of the variation in workout strategy occurs across lenders would suggest that the institutional factors are more significant.

The model developed here can be extended in several potentially important ways. For example, the restriction to a fixed dividend payout and fixed property value volatility can be relaxed, making the borrower's problem a dynamic stochastic control problem. It is likely that the borrower's incentive to extract cash or adjust risk will vary with current loan-to-value ratio and other operating variables. The model can also be extended to include the original loan terms as endogenous variables and study how workout strategies influence original loan pricing.

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Appendix A

Valuation of the Wedge

Because the symbol W has been used in the text to denote the market value of a particular claim, $B(s)$ will be used in the appendixes to denote standard Brownian motion under a particular filtration, F_t. The standard expectation notation $E[v]$ denotes the expectation under the true probability measure. $E_Q[v]$ denotes the expectation under the equivalent risk-neutral probability measure, Q. The following derivation is based on the result that the value of a contingent claim is equal to the expected discounted value of the contingent claim's payoff, discounting at the risk-free rate, when the expectation is taken with respect to the equivalent risk-neutral probability measure Q (Cox, Ingersoll and Ross 1985).

The payoff pattern to be valued is

$$[(1-\gamma(V(T+1)]^- \equiv \begin{Bmatrix} 0 & \text{if } V(T+1) > L \\ (1-\gamma)V(T+1) & \text{if } V(T+1) \le L \end{Bmatrix}. \tag{A.1}$$

Under the assumptions identified in the section "General Assumptions" and following the standard change of measure arguments, its price at time T is

$$E_Q\{e^{-rt}[(1-\gamma)V(T+1)]^-\}, \tag{A.2}$$

where the expectation is taken under the equivalent martingale (*i.e.*, the risk-neutral) measure, Q.

With no cash dividend, $V(s)$ follows a lognormal process with mean μ and standard deviation σ_V. Therefore, we can write

$$V(T+1) = V(T)e^{(\mu-(\sigma^2/2))+\sigma_V(B(T+1)-B(T))}. \tag{A.3}$$

After a change in measure and applying the Cameron–Martin–Girsanov Theorem, (A.3) becomes

$$V(T+1) = V(T)e^{(r-(\sigma^2/2))+\sigma_V(B^*(T+1)-B^*(T))}. \tag{A.4}$$

B^* represents Brownian motion under the equivalent measure, Q.

From (A.1) and (A.2),

$$W(T) = \int_{-\infty}^{\infty} \left[(1-\gamma)V(T)e^{(r-(\sigma^2/2))+\sigma_V y}\right]^- e^{-r}\left(\frac{1}{\sqrt{2\pi}}e^{-y^2/2}\right)dy. \tag{A.5}$$

Next, define

$$y^* \equiv \left(\frac{1}{\sigma_V}\right)\left[\ln\left(\frac{L}{V(T)}\right) - \left(r - \frac{\sigma_2^v}{2}\right)\right]. \tag{A.6}$$

Then

$$W(T) = \left(\frac{(1-\gamma)V(T)}{\sqrt{2\pi}}\right)\int_0^{y^*} e^{-((y-\sigma_V^2)/2)}dy = (1-\gamma)\Phi(-z_1)V(T), \tag{A.7}$$

where

$$z_1 = \frac{\ln\left(\frac{V(T)}{L}\right) + \left(r + \frac{\sigma_V^2}{2}\right)}{\sigma_V}, \tag{A.8}$$

and $\Phi(z)$ represents the cumulative Normal distribution function.

Appendix B

Proof that $V^* > L$

If the borrower offers the lender a payment b equal to the lender's indifference payment, then the borrower's payoff from default and extension is $C - b$. Substituting for b using Equations (5) and (9),

$$C - b = C - (\gamma - 1)V(T) - C - W = (1-\gamma)V(T) - W. \tag{B.1}$$

V^* is defined as the value of V at which the payoff from default and extension is just equal to the payoff from repaying the loan. Setting $(C - b) = V(T) - L$, and rearranging and substituting for W using Equation (3),

$$V^*(T) = \frac{L}{(\gamma + (1-\gamma)\Phi(-z_1))}. \tag{B.2}$$

Although $V(T)$ is included in the definition of z_1, it clear that $V^*(T) > L$ because the cumulative normal function will always be less than one making the denominator less than one.

Note also that expression (B.1) combined with the definition of W in (A.7) shows that

$$(C - b) = (1-\gamma)\Phi(z_1)V(T), \tag{B.3}$$

confirming that $(1-\gamma_M)V(T)$ (for $\gamma_M > \gamma_P$) can be used as a lower bound on the payment to the borrower in the default region.

Appendix C

Valuation of Borrower Payoff Patterns and Their Derivatives

When valuing the borrower's expected payoff at loan maturity, there are two differences from the valuation problem in Appendix A. First, the time difference is longer than one period; second, the borrower receives some payoff over the full support of the asset distribution's pdf. Denoting the value of the borrower's claim on the asset at loan maturity T as C and the right to receive dividends during the term of the loan as R, the borrower's portfolio is given by

$$P = C + R. \tag{C.1}$$

As in Appendix A, the market value of this portfolio is given by the expectation of the discounted future cash flows from each component where the expectation is taken under the equivalent martingale measure, Q. Again assuming a complete market, we have

$$\begin{aligned} P(0) = &\int_{-\infty}^{y^*} (1-\gamma_i)V(0)e^{(r-\xi-(\sigma^2/2))\tau+\sigma_V y} f(y)e^{-r\tau}\,dy \\ &+ \int_{y^*}^{\infty} \left[\left(V(0)e^{(r-\xi-(\sigma^2/2))+\sigma_v y}\right) - L\right]e^{-r\tau} f(y)\,dy + V(0)[1-e^{-\xi\tau}] \end{aligned} \tag{C.2}$$

where

$$y^* = \frac{1}{\sigma}\left[\ln\left(\frac{L}{V(0)\gamma_i}\right) - \left(r - \xi - \frac{\sigma^2}{2}\right)\tau\right]$$

and f(y) is the standard normal density function.

The two integrals in (C.2) are truncated lognormal expectations, and so

$$P(0) = (1 - \lambda_i)V(0)e^{-\xi\tau}\Phi\left(-z_1^i\right) + V(0)e^{-\xi\tau}\Phi\left(z_1^i\right) \\ - Le^{-r\tau}\Phi\left(z_2^i\right) + V(0)\lfloor 1 - e^{-\xi\tau}\rfloor. \tag{C.3}$$

Given the closed form expression for the borrower's portfolio, calculating the partial derivatives with respect to ξ and σ_V is straightforward.[39]

[39] A useful fact for calculating these derivatives is that $\frac{\partial\Phi(z)}{dx} = n(z)\frac{\partial z}{\partial x}$.