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A Brief Note on Effective Duration-Volatility Relationship

Because of the increased likelihood of low interest rate-high prepayment rate paths occurring when volatility increases, on the surface, it can appear that effective durations would shorten as volatility increases. However, the situation is more complicated, because, for example, higher volatility also increases the probability of high-interest-rate, slow-prepayment rate paths.

According to the "OAS to Swaps Report" in manifold MB733, for the close of Friday, January 12, 2001, conventional 7s had an effective duration of 3.2 assuming implied volatilities and 3.3 assuming fixed historical volatilities. These durations increased by 0.3 and 0.4, respectively, from the durations for the previous day. With volatility having declined roughly 1% from Thursday to Friday (and rates having risen about 12bp), the view that decreasing volatility should tend to increase durations and hence, implied-volatility duration should increase more than fixed-volatility duration, is in opposition to these computed values.

In Terms of Volatility Durations

One approach to better understanding how effective durations might increase as volatility rises is to look at volatility durations. For example, the "Effective/Empirical Duration Report" (manifold MB733 contains a swaps version of the Treasury-based version on manifold MB728) can be used as a guide. For the close of January 12, conventional 30-year 6.5s, 7s, and 7.5s had volatility durations of 0.34, 0.30, and 0.27, respectively. These figures roughly follow a pattern similar to that of simpler equity options — sensitivity to volatility peaks near the strike price, which for mortgages often corresponds to a price somewhere near par. So, if we think of 6.5s as the current coupon (roughly corresponding to the mortgage "strike" price), then they should have the highest volatility duration. If our intuition can accept this relationship, then the explanation for the duration changes follows if one assumes that 7s in a +50bp scenario behave like 6.5s in the base case (in other words, that the volatility duration of 7s increases as rates rise), for example.

Assuming a constant OAS to swaps (-3bp) and using implied volatilities for the close of January 12, one can derive the matrix of scenario prices for conventional 7s shown in Figure 28 (full prices are shown).[6]

Figure 28. Volatility and Rate Shift Scenario Prices for Conventional 7s

	-50 bp Rate Shift	Base	+50 bp Rate Shift	Effective Duration
-1% Vol	102.098	100.861	99.062	3.01
Base Implied Vols	101.888	100.556	98.720	3.15
+1% Vol	101.682	100.252	98.368	3.30
Volatility Duration	0.20	0.30	0.35	

Source: Salomon Smith Barney.

[6] The prices for each of the two volatility shift scenarios are obtained by shifting the implied volatilities for the close of January 12, recalibrating the term structure model, and then calculating prices assuming unchanged OAS.

Thus, the "+1% Vol" case implies an effective duration of (101.682 - 98.368)/100.252 = 3.30, while the base-case implied volatilities give an effective duration of (101.888 - 98.720)/100.556 = 3.15.

And we now see that the *longer volatility durations as rates increase* pattern results in longer effective durations on conventional 7s as volatility increases.[7] (However, this would not be the case if the volatility durations moved in the opposite direction as they do for deep discounts. In fact, the more traditional "as volatility increases, effective duration shortens" pattern should be expected for deep discounts.) In terms of hedging, those investors using implied-volatility-model effective durations should guard against extending durations (at least for 7s) in a rising volatility scenario.